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Can Lead–Lag Guidance Compensator Guarantee Zero Miss Distance?

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I. Introduction

IN A series of papers [1–4], it was claimed that a lead–lag precompensator in the proportional navigation loop can force zero miss distance (ZMD) and avoid saturation against *any bounded target maneuver*. We use a recent approach to worst-miss distance [5] and demonstrate that the lead–lag approach is sensitive and renders miss distance far from zero.

Consider a linearized proportional navigation (PN) loop in planar motion, as described in Fig. 1. In this figure, $x_1 \triangleq \Delta y$ is the target-missile separation normal to the line of sight (LOS); $\Delta\ddot{y} = \mathrm{d}^2(\Delta y)/\mathrm{d}t^2$, $x_2 = \dot{x}_1$, σ is the LOS angle relative to an inertial reference line; V_c is the closing speed; N' is the navigation gain; u and v are the missile and target actual accelerations, respectively, normal to LOS; t_f is the final (terminal) time; and $t_{\mathrm{go}} = (t_f - t)$ is the time to go. Note that with R as the range, $V_c = R/t_{\mathrm{go}}$. In the first paper [1], the abstract states the following:

> A novel approach based on input-output stability renders design guidelines that assure operation in the nonsaturating region given the missile-target maneuver ratio. These guidelines yield a proportional-navigation-based guidance law that assures zero miss distance for any bounded target maneuver. It is shown that if the total dynamics of the guidance loop are designed to be positive real, and the effective proportional navigation constant is chosen to be a simple function of the maneuver ratio, no saturation shall occur. The illustrative examples validate the analysis and show that the new guidance law is robust enough to guarantee a significant performance improvement, even if the design guidelines are somewhat loosened.

Later, the Conclusions section in [1] reads as follows:

> This paper presented a simple improvement to the well-known proportional navigation guidance (PNG) law that assured that saturation of maneuver acceleration or other limited state variables was prevented. Based on previous studies and an illustrative example, it was shown that preventing saturation yields ZMD. The improved guidance, called ZMD–PNG, was based on the assumption that the target maneuver was bounded.

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In [1] (p. 697), the target maneuver is characterized using the words, "any target maneuver with bounded maximal value." In mathematical terms, [1] (p. 697) characterizes the target maneuver a_T using the relation $a_T \in L^\infty[0, t_f]$. According to [1–4], the guidance improvement is achieved using the navigation gain

$$N' \geq 2\mu/(\mu - 1) \tag{1}$$

where μ is the missile-target maximal acceleration ratio. The precompensator [see [1], Eq. (56)]

$$K(s) = \prod_{i=1}^{m}(1 + \tau_{zi}s) \tag{2}$$

consists of proportional-derivative (PD) elements such that the augmented transfer function $G(s)$ is biproper, and the overall guidance loop transfer function $H(s) = G(s)/s$ is positive real (PR), where $G(s) = G_M(s)K(s)G_s(s)$. These design guidelines have two conceptual steps. First, N' in Eq. (1) and the PR property of $H(s)$ are responsible for nonsaturation. Now that the guidance loop is linear, the biproperness of $G(s)$ is responsible for ZMD. From [1] (p. 697), "if the saturations of the guidance loop remain inactive, then if the degree of the numerator of $G(s)$ equals the degree of the denominator of $G(s)$, ZMD will be obtained for any flight time and any bounded target maneuver." However, because PD elements magnify noise, it was suggested [see [1], Eq. (57)] to replace the preceding PD compensator $K(s)$ by the lead–lag action

$$K(s) = \prod_{i=1}^{m}\frac{(1 + \tau_{zi}s)}{(1 + \tau_{pi}s)} \tag{3}$$

It was claimed [1] (p. 698) that although now $H(s) \notin PR$, "nontheless, if the additional lag is not too large, we can still prevent acceleration saturation." At this stage, several questions arise with respect to [1–4]:

1) What is "not too large" in terms of the additional lag to prevent saturation?

2) Quantitatively, how much miss distance does the additional lag contribute against the worst-bounded target maneuver?

Unfortunately, these crucial questions are not addressed in [1–4]. Thus, one is left to analyze the numerical examples in [1–4]. It should be emphasized that due to the any bounded target maneuver claim in [1–4], one needs a way to calculate the worst-case PN, that is, the miss distance in PN against the worst-bounded target maneuver. To this end, let us first review some relevant results.

II. Worst-Case Proportional Navigation

To study the effect of the worst-target maneuver with respect to miss distance, we assume here that the missile is guided by PN and that the target maneuver is constrained by

$$|v| \leq \rho_v \tag{4}$$

The term "worst-case maneuver" needs an explanation. Although this maneuver is the worst from the missile stand point, it is the best from the target stand point—it maximizes the miss distance. According to [5], Fig. 9.21, the worst-case miss distance is the steady-state value of $m(t_{\mathrm{go}})$, where $m(t_{\mathrm{go}})$ is defined in Fig. 2, and t_{go} is time to go. The worst-target maneuver v^* is also given in Fig. 2.

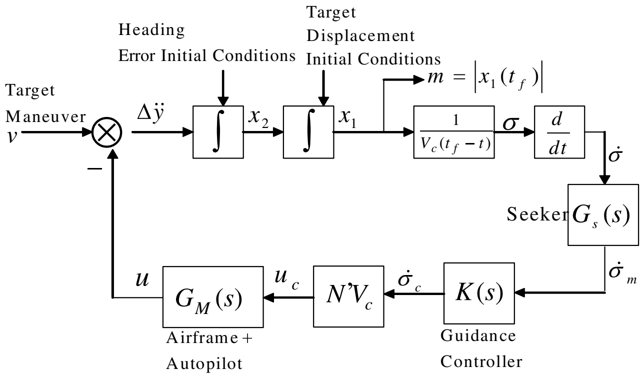

Fig. 1 Proportional navigation guidance loop.

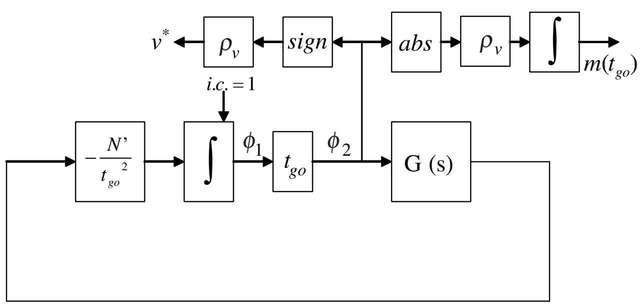

Fig. 2 Worst-case miss distance and target maneuver.

Using Simulink to mechanize the diagram of Fig. 2 enables us to easily calculate the worst-miss distance and the worst-target maneuver. The interested reader will find in [5], Chapter 9, Section F.3 the detailed derivation. Although the derivation in [5] is a special case of differential game; note that one can derive this result directly from optimal control. Moreover, one may derive the same result from adjoint analysis. Indeed, the closed loop in Fig. 2 is equivalent to the adjoint loop, and the absolute value operation "abs" adds to the miss distance the contribution of each "peak" in the adjoint (sensitivity) function.

III. Two Examples

As mentioned earlier, [1–4] do not study quantitatively the effect of controller (3) on the miss distance. Moreover, in spite of their claim regarding any bounded target maneuver, their numerical examples do not use the worst-target maneuver. Rather, [1–4] apply numerically constant, sinusoidal, and random maneuvers. In this section, we revisit two examples from [1–4] and apply worst-target maneuvers with the aid of the block diagram in Fig. 2.

A. Example 1

This example follows the example in [1]. Let

$$G(s) = \frac{1}{1+0.3s}\cdot\frac{100}{s^2+10s+100} \tag{5}$$

the missile-target maneuver ratio $\mu = 2$, which according to Eq. (1) implies $N' \geq 4$, and $\rho_v = 1$ g. Gurfil et al. [1] suggests to use the guidance controller

$$K(s) = \prod_{i=1}^{3}\frac{(1+0.23s)}{(1+0.05s)} \tag{6}$$

Using Simulink to mechanize the diagram of Fig. 2 with $N' = 4$, let us construct in Fig. 3 the function $m(t_{\mathrm{go}})$. Using $m = \lim_{t_{\mathrm{go}}\to\infty} m(t_{\mathrm{go}})$, the worst-miss distance becomes $m = 2.65$ m. Likewise, Fig. 4 presents the worst-target maneuver v^*. Decreasing the navigation gain to the value $N' = 3$, the miss distance reduces to $m = 1.08$ m. To conclude, note the following:

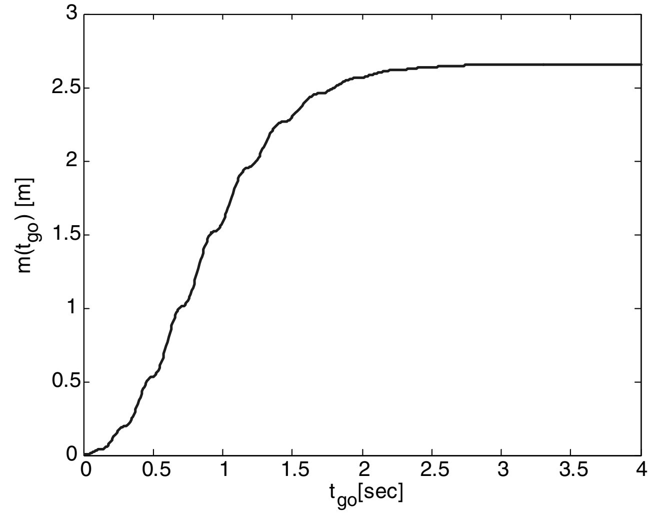

Fig. 3 Worst-miss distance in example 1.

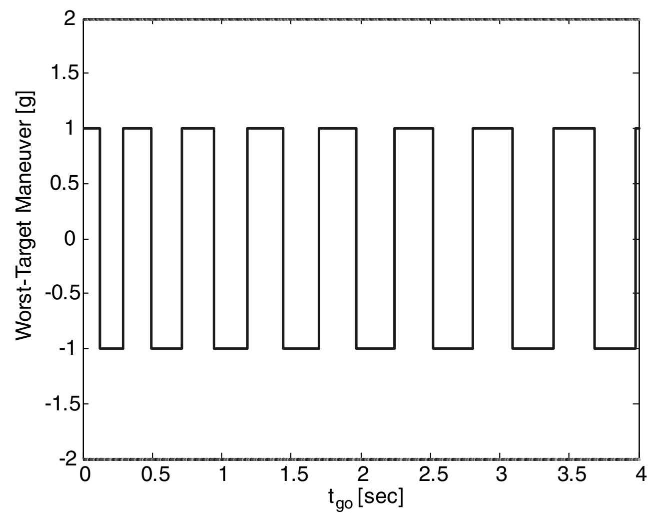

Fig. 4 Worst-target maneuver in example 1.

1) For the standard PN [$K(s) = 1$], the worst-miss distance becomes $m = 3$ m. Thus, with $K(s)$ of Eq. (6), only a small miss improvement is achieved.

2) The suggested $K(s)$ generates a large miss ($m = 2.65$ m) that cannot be described as ZMD.

3) $N' = 3$ is superior to $N' = 4$ as far as the worst miss is concerned.

4) As can be seen from Fig. 4, the worst-target maneuver v^* is much different from constant, sinusoidal, or random function. One may argue that this v^* is not practical for implementation. However, it is emphasized that papers [1–4] deal with an ideal target and any bounded target maneuver, $|v| \leq \rho_v$. This claim means that the target maneuver can be *any piecewise continuous* maneuver, which is certainly satisfied by the maneuver described in Fig. 4.

5) The preceding results can be extended to nonideal targets. This is not done here because the target transfer function is not specified in [1–4]. Nevertheless, in the next example we will demonstrate an evasive maneuver for a nonideal target.

B. Example 2

This example is taken from [4]. A nonminimum phase 13th order missile is truncated using balanced realization to obtain

$$G(s) = \frac{1}{1+0.1s}\cdot\frac{-s/40.3+1}{(1+s/23.3)(1+s/1.93)} \tag{7}$$

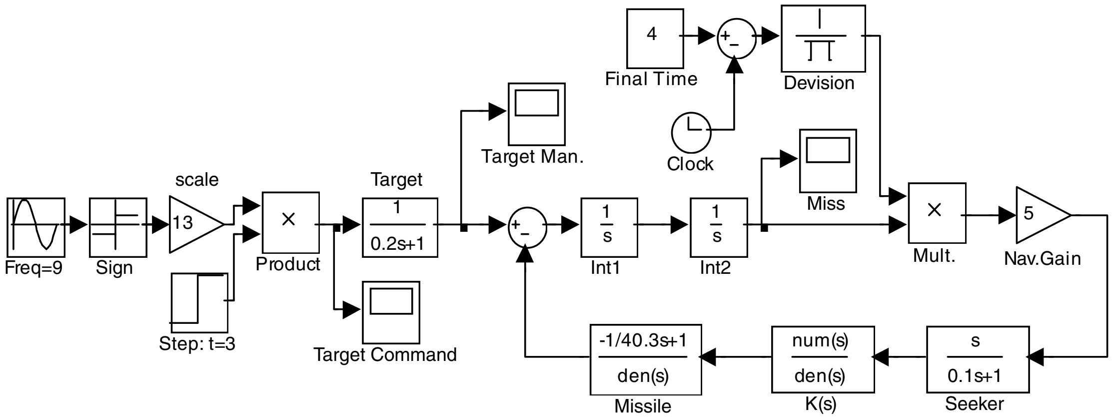

Fig. 5 Simulink construction of suboptimal evasive maneuver in example 2.

At this stage the nonminimum phase property is preserved. It should be emphasized that a nonminimum phase missile does not satisfy the positive-real condition. Nevertheless, this transfer function is further truncated into

$$G(s) = \frac{1}{1+0.1s} \cdot \frac{1}{1+0.56s} \tag{8}$$

Now, the nonminimum phase property is no longer preserved. Based on model (8), it was suggested in [4] to apply $N' = 5$ with guidance controller

$$K(s) = \frac{0.203s^2 + 0.387s + 1}{0.0001s^3 + 0.038s^2 + 0.087s + 1} \tag{9}$$

First, we wish to comment on approximation (8). At a first glance, based on a 4 s step response (see Fig. 5), it seems like Eq. (8) approximates Eq. (7) quite well. However, in a small scale step response (0.3 s), the approximation is poor. Indeed, we can apply the preceding $K(s)$ to both Eqs. (7) and (8). Using the block diagram in Fig. 2 to calculate the worst-miss distance against PN, it turns out that for $N' = 5$, in the second order approximation (8) the worst-miss distance is $m = 3.49$ m, whereas with the third order approximation (7) it becomes $m = 18.22$ m. For $N' = 3$, in the second order approximation (8) the worst-miss distance is $m = 1.33$ m, whereas with the third order approximation it becomes $m = 3.87$ m. We conclude that as far as the worst-miss distance is concerned, both approximations are far from ZMD, and that from the missile standpoint, $N' = 3$ is superior to $N' = 5$.

The worst-target maneuvers mentioned earlier are based on Fig. 2. Now, let us construct a simple suboptimal evasive maneuver that induces large miss distance. To this end, refer to the Simulink scheme presented in Fig. 5, with $N' = 5$ as in [4]. First, recall Fig. 1 with the miss distance calculated by $m = |x_1(t_f)|$. As can be seen from Fig. 5, the evasive maneuver is formed of a square wave command injected to a target transfer function. We consider two cases. In the first, the target is ideal. Here, it is found that for flight time $t_f = 4$ s and a target maneuver starting at $t = 1$ s (3 s before termination), a sine wave of frequency 7 rad/s and a scale factor of 10 m/s^2, generate the miss distance $m = 8.28$ m. In the second case, we go beyond the ideal target that was assumed in [1–4] and assume a target with a single time constant set to 0.2 s. Here, it is found that for flight time $t_f = 4$ s and a target maneuver starting at $t = 3$ s (1 s before termination), a sine wave of frequency 9 rad/s, and a scale factor of 13 m/s^2, generate the miss distance $m = 1.17$ m. Although less than the worst-case 18.22 m reported earlier, the preceding miss distances are sufficiently large to convince the reader that the approach of [1–4] does not guarantee ZMD.

IV. Conclusions

To achieve the PR property it has been proposed to insert in the guidance loop a linear compensator formed of PD elements together with a noise filter. It has been shown in the present paper that this leads to sensitivity with respect to the worst-target miss. There are several drawbacks in the proposed approach. First, it contains no analysis concerning the miss distance in case a missile deviates from the PR property. Instead, unproved claims, such as, "nontheless, if the additional lag is not too large, we can still prevent acceleration saturation," are used. Second, it does not mention explicitly that the PR phase condition cannot be met in the case of nonminimum phase missiles. Third, according to the proposed approach, N' has a lower bound. However, using two examples, the present note has shown that from worst-miss distance, N' should be smaller than predicted.

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